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$$2ab(b\pm a),\quad 2ab(a\pm b),\quad \mp 2ab(a\pm b)^2/c.$$

From the first two coördinates we see that the axes of these parabolae are the bisectors of the angle at $\alpha\beta$.

Problem 204 was also solved by D. B. Northrup, Schenectady, N. Y.

205. Proposed by **L. C. WALKER, A.M.**, Professor of Mathematics, Colorado School of Mines, Golden, Col.

Having given any two systems of conjugate semi-diameters of an ellipsoid, the parallelopiped which has any three for continuous edges is equal to that which has the other three for continuous edges.

Remark by **H. B. LEONARD, A.B.**, Chicago, Ill.

Solved in C. Smith's "An Elementary Treatise on Solid Geometry," page 76.

CALCULUS.

166. Proposed by **T. N. HAUN**, Mohawk, Tenn.

Find the volume of the solid formed by the revolution of the curve $(y^2+x^2)=a^2(x^2-y^2)$ round the axis of x.

II. Solution by **G. W. GREENWOOD, B. A.** (Oxon), Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.

This equation represents singly the two lines $y=\pm x\sqrt{(a^2-1)/(a^2+1)}$, and the surface generated is simply a right circular cone whose area, volume, bounded by the plane $x=c$, is simply $\frac{1}{3}\pi c^3(a^2-1)/(a^2+1)$, and where $a\neq 1$.

168. Proposed **F. P. MATZ, Sc. D., Ph. D.**, Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

The tangent of what Cartesian curve makes an x-intercept always m times as long as as the corresponding y-intercept?

II. Solution by **G. W. GREENWOOD, B. A.** (Oxon), Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.

Let the tangent at P (x, y) meet the axes in the points A and B. Let the perpendicular from P to OY meet it at M. Call θ the angle APM.

Then $PB=m.PA$, $OM=m.AM$; *i. e.*, $y=m.PM.\tan\theta=mx(dy/dx)$.

$\therefore y^m=cx$.

AVERAGE AND PROBABILITY.

145. Proposed by **F. P. MATZ, Sc. D., Ph. D.**, Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

In each quadrant of a given circle, a circle is described at random. A point is taken at random in each of these circles. What is the average area of the quadrilateral formed by joining with straight lines these four points?

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, West Va.

Let P, Q, R, S be the centers of the four circles. The average area of the required quadrilateral is the same as the average area of the quadrilateral formed by joining the centroids of the four circles, or their centers.

Let $OP=s$, $PC=t$, $OQ=u$, $QF=v$, $OR=w$, $RE=x$, $OS=y$, $DS=z$, $OG=a$, $\angle AOP=\theta$, $\angle AOQ=\phi$, $\angle BOR=\psi$, $\angle BOS=\rho$

The limits of t, v, x, and z are 0 and $a(\sqrt{2}-1)=m$; of s, $t\sqrt{2}$ and $a-t$; of u, $v\sqrt{2}$ and $a-v$; of w, $x\sqrt{2}$ and $a-x$; of y, $z\sqrt{2}$ and $a-z$; of θ, $\sin^{-1}(t/s)=\theta''$ and $\frac{1}{2}\pi-\theta''=\theta'$; of ϕ, $\sin^{-1}(v/u)=\phi''$ and $\frac{1}{2}\pi-\phi''=\phi'$; of ψ, $\sin^{-1}(x/w)=\psi''$ and $\frac{1}{2}\pi-\psi''=\psi'$; of ρ, $\sin^{-1}(z/y)=\rho''$ and $\frac{1}{2}\pi-\rho''=\rho'$.

The area of $PQRS=\frac{1}{2}[su\sin(\theta+\phi)+uw\sin(\phi+\psi)+wy\sin(\psi+\rho)+sy\sin(\theta+\rho)]=A$.

$$\therefore \Delta=\frac{\int_0^m\int_0^m\int_0^m\int_0^m\int_{t\sqrt{2}}^{a-t}\int_{v\sqrt{2}}^{a-v}\int_{x\sqrt{2}}^{a-x}\int_{z\sqrt{2}}^{a-z}\int_{\theta''}^{\theta'}\int_{\phi''}^{\phi'}\int_{\psi''}^{\psi'}\int_{\rho''}^{\rho'}A\,I}{\int_0^m\int_0^m\int_0^m\int_0^m\int_{t\sqrt{2}}^{a-t}\int_{v\sqrt{2}}^{a-v}\int_{x\sqrt{1}}^{a-x}\int_{z\sqrt{2}}^{a-z}\int_{\theta''}^{\theta'}\int_{\phi''}^{\phi'}\int_{\psi''}^{\psi'}\int_{\rho''}^{\rho'}I}$$

where $I=dt\,dv\,dx\,dz\,sds\,udu\,wdw\,ydy\,d\theta\,d\phi\,d\psi\,d\rho$.

$$\Delta=\frac{6^8}{(3\pi-28+16\sqrt{2})^4a^{12}}$$

$$\times\int_0^m\int_0^m\int_0^m\int_0^m\int_{t\sqrt{2}}^{a-t}\int_{v\sqrt{2}}^{a-v}\int_{x\sqrt{2}}^{a-x}\int_{z\sqrt{2}}^{a-z}\int_{\theta''}^{\theta'}\int_{\phi''}^{\phi'}\int_{\psi''}^{\psi'}\int_{\rho''}^{\rho'}A\,I$$

$$\Delta=\frac{648}{(3\pi-28+16\sqrt{2})^2a^6}\Big[\int_0^m\int_{t\sqrt{2}}^{a-t}\int_{\theta''}^{\theta'}\int_0^m\int_{v\sqrt{2}}^{a-v}\int_{\phi''}^{\phi'}$$

$$\times s^2u^2\sin(\theta+\phi)dt\,ds\,d\theta\,dv\,du\,d\phi$$

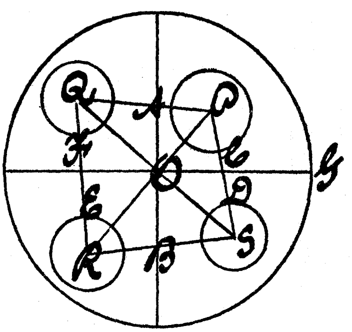
$$+\int_0^m\int_{v\sqrt{2}}^{a-v}\int_{\phi''}^{\phi'}\int_0^m\int_{x\sqrt{2}}^{a-x}\int_{\psi''}^{\psi'}$$

$$\times u^2w^2\sin(\phi+\omega)dv\,du\,d\phi\,dx\,dw\,d\psi$$

$$+\int_0^m\int_{x\sqrt{2}}^{a-x}\int_{\psi''}^{\psi'}\int_0^m\int_{z\sqrt{2}}^{a-z}\int_{\rho''}^{\rho'}$$

$$\times w^2y^2\sin(\psi+\rho)dx\,dw\,d\psi\,dz\,dy\,d\rho$$

$$+\int_0^m\int_{t\sqrt{2}}^{a-t}\int_{\theta''}^{\theta'}\int_0^m\int_{z\sqrt{2}}^{a-z}\int_{\rho''}^{\rho'}s^2y^2\sin(\theta+\rho)dt\,ds\,d\theta\,dz\,dy\,d\rho\Big]$$



$$=\frac{9(3729-2624\sqrt{2})a^2}{25(3\pi-28+16\sqrt{2})^2}=\left(\frac{3\,a}{5}\right)^2\left(\frac{32\sqrt{2}-41}{3\pi-28+16\sqrt{2}}\right)^2=\tfrac{2}{5}a^2, \text{ nearly,}$$

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PROBLEMS FOR SOLUTION.

ALGEBRA.

187. Proposed by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

Express by radicals the roots of $x^7+px^5+\frac{2}{7}p^2x^3+\frac{1}{49}p^3x+r=0$.

188. Proposed by GUY SCHUYLER.

$xy+ab=2ax,\quad x^2y^2+a^2b^2=2b^2y^2$

GEOMETRY.

209. Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, England.

Find by a geometrical method the maximum value of $\sin\theta\cos\theta\cos2\theta$.

210. Proposed by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

Let *ADC* be a triangle with angle $C=120°$, and let the interior bisector of angle *C* meet *AD* in *B*. Prove that $2.CB$ is the harmonic mean between *CA* and *CD*.

211. Proposed by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

Prove the validity of the following construction of an inscribed regular pentagon and regular decagon: Draw any two perpendicular radii of the given circle with center *C*. Call *E* the end of one radius *CE* and *M* the middle point of the perpendicular radius *CM*. Take the point *R* on *CM* produced through *C* such that $RCM=EM$. Then RC=side of inscribed regular decagon, RE=side of inscribed regular pentagon.

CALCULUS.

172. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

Solve $x\dfrac{dy}{dx}=\dfrac{y}{y^{-1}-\log x}$.

DIOPHANTINE ANALYSIS.

119. Proposed by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

If p be any prime number and n any positive integer, the congruence